Journal of Economic Integration
25(3), September 2010; 548-568

Collective Pegging to an External Currency: Lessons from a Three-Country Model

Chrysost Bangaké*
Orleans University

Jean-Baptiste Desquilbet
Arois University

Nabil Jedlane
Cadi Ayyad University (UCAM)

Abrstract

This paper examines the circumstances under which it is beneficial for small countries in a currenct union to peg their currency to a large one (euro zone for example). For these purposes, we provide a three-country theoretical model extending the two-country model by Ricci (2008). The theoretical model is based on a Ricardian model of free traded, with specialised economies each producing one traded and one-traded good. We show that when the home country belongs in a monetary union and its exchange rate is anchored to the large country, the stability of its economy depends on the variability of real and monetary shocks for the large country. Furthermore, if the monetary rule in the currency union is higher than the average rate of growth of money supply of large country or if it is difficult to find a monerary rule in the currency union, it is advantagesous to

*Corresponding address: Chrysost Bangaké, Laboratoire d'Economie d'Orléans, CNRS, UMR 6221 Faculté de Droit, d'Economie et de Gestion, rue de Blois B.P 6739-45067 Oréans Cedex 2 (France), Phone: +33(0) 6 25 81 43 96 Office phone: +33 (0) 2 38 41 70 37, e-mail: chrysost.bangake@univ-orleans.fr. / Jean-Baptiste Desquilbet, Université d' Artois, UFR EGASS, 9 Rue du Temple, B.P 10665-62030, Arras Cedex, France, e-mail: jbaptiste.desquilbet@univ-artois.fr, / Nabil Jedlane, Université de Cadi Ayyad (UCAM), Ecole Nationale de Commerce et de Gestion Avenue Allal El Fassi, B.P 3748 Amerchich, Marrakech, Maroc, e-mail: nabil.jedlane@gmail.com

anchor the single currency to that of the large country.

• **JEL Classification:** E42, E52, F02, F36, F4

• **Key Words:** Optimum Currency Area, Currency Union, Cost-Benefit Analysis, Collective Pegging, Small Countries, ECOWAS.

I. Introduction

Since the successful launch of the euro zone, there has been a renewed and a growing interest in monetary union around the world. One example is a project to create a common currency among 15 countries of Economic Community of West African States (ECOWAS). The particularity of this project is that the region includes few countries members of the West African Economic and Monetary Union (WAEMU)[1] whereby exchange rates are collectively pegged to euro and non WAEMU members of ECOWAS with heterogeneous monetary regimes (managed floating, crawling peg). On April 2000, the non-WAEMU countries declared their intention to proceed to a second monetary union to be known as the West African Monetary Zone (WAMZ).[2] This would be a first step toward a wider monetary union in all ECOWAS.

Given the size and the economic structures of these countries (small countries) such a project raises the question of external anchor currency. Although this question has been highly interesting, there is however a glaring paucity of theoretical and empirical work except the model of Braga de Macedo (1985) which analyzed the interaction of the small countries in monetary unions with France. This paper tries to fill this gap.

As our main contribution, we examine the circumstances under which it is beneficial for small countries in a currency union to peg their currency to a large one (euro zone for example). For that purpose, we develop a three-country theoretical model extending the two-country model by Ricci (2008). The theoretical model is based on a Ricardian model of free trade, with specialised economies each producing one trade and one non-traded good. It includes both monetary and real arguments considered in the optimum currency area literature. Such an approach shows that when the home country belongs in a monetary union

[1]Members (Benin, Burkina Faso, Cote d'Ivoire, Guinea-Bissau, Mali, Niger, Senegal, and Togo) use the CFA franc issue by their bank, the BCEAO

[2]Its prospective members include the Gambia, Ghana, Guinea, Nigeria, and Sierra Leone.

and its exchange rate is anchored to the large country, the stability of its economy depends on the variability of real and monetary shocks for the large country. Furthermore, if the monetary rule in the currency union is higher than the average rate of growth of money supply of large country or if it is difficult to find a monetary rule in the currency union, it is advantageous to anchor the single currency to that of the large country.

The rest of paper is organized as follows. The section II presents the model. While section III examines shocks and adjustments, section IV measures and discusses the expected net benefits from the participation of small countries (West African countries) in a currency union but also other kinds of exchange rate regimes. We explore the question of external arrangement that might be appropriate for small countries. Section V dresses some concluding remarks.

II. The Model

We develop a three-country theoretical model extending the two-country model by Ricci (2008). It is a Ricardian model of free trade, with specialised economies each producing one traded and one non-traded good. The analysis is static and ignores the existence and accumulation of capital. Labour is fully mobile between sectors within the same country but immobile between countries. Money is taken into account through standard Cambridge equations. Inflation occurs whenever the demand for goods rises above full capacity, unemployment whenever the demand for goods falls below full capacity. Exchange rates are determined by trade balance equilibrium in flexible exchange rate regimes, whereas any kind of fixed exchange regime puts some constraints on the money supply. The two small countries labelled countries 1 and 2, while the large one is country 3.

A. Markets for Goods

The traded goods denoted A, B and C and nontraded goods N_1, N_2 and N_3 are produced respectively by countries 1, 2 and 3. The three countries are endowed with fixed quantities of labour, denoted L_1, L_2 and L_3. Production functions exhibit constant returns to scale, which is consistent with complete specialisation.

Supplies of goods are given by

$$A^s = \theta_A L_A, B^s = \theta_B L_B, C^s = \theta_C L_C, N_1^s = \theta_{N1} L_{N1}, N_2^s = \theta_{N2} L_{N2}, N_3^s = \theta_{N3} L_{N3} \tag{1}$$

with $L_A + L_{N1} \leq L_1$; $L_B + L_{N2} \leq L_2$; $L_C + L_{N3} \leq L_3$
where θ_i and L_i denote respectively the average (and marginal) labour productivity and the employment level in sector i.

Firms behave competitively. The zero-profit condition in each sector implies that the real wage is equal to labour productivity

$$\frac{w_1}{P_A} = \theta_A, \ \frac{W_1}{P_{N1}} = \theta_{N1}, \ \frac{w_2}{P_B} = \theta_B, \ \frac{W_2}{P_{N2}} = \theta_{N2}, \ \frac{w_3}{P_C} = \theta_C, \ \frac{W_3}{P_{N3}} = \theta_{N3} \quad (2)$$

where w_1 w_2 and w_3 are nominal wages in countries 1, 2 and 3, respectively, and P_i is the price of goods in sector i (in the currency of the employers' or producers' country).

Consumers have Cobb-Douglas preferences over the three traded goods (A, B and C) and their home country nontraded good (N_1 or N_2 or N_3). Preferences are assumed to differ in the three countries in order to investigate the effects of the degree of openness and of the symmetry of shocks on the desirability of a currency union.

The representative consumer k of country 1 maximises the following utility function

$$U_k = A_k^{\alpha_1} B_k^{\beta_2} C_k^{\gamma_1} N_{1k}^{1-\alpha_1-\beta_1-\gamma_1} \text{ subject to } P_A A_k + \frac{P_B}{e_1}\tau_1 B_k + P_C e_3 \tau_3 C_k + P_{N1} N_{1k} = Y_k \quad (3)$$

Likewise, the representative consumer m of country 2 maximises

$$U_m = A_m^{\alpha_2} B_m^{\beta_2} C_m^{\gamma_2} N_{2m}^{1-\alpha_2-\beta_2-\gamma_2} \text{ subject } P_A e_1 \tau_1 A_m + P_B B_m + P_C e_2 \tau_2 C_m + P_{N2} N_{2m} = Y_m \quad (4)$$

and the representative consumer n of country 3 maximises

$$U_n = A_n^{\alpha_3} B_n^{\beta_3} C_n^{\gamma_3} N_n^{1-\alpha_3-\beta_3-\gamma_3} \text{ subject to } \frac{P_A}{e_3}\tau_3 A_n + \frac{P_B}{e_2}\tau_2 B_n + P_C C_n + P_{N3} N_{3j} = Y_n \quad (5)$$

where e_1, e_2 and e_3 are respectively the nominal exchange rates between countries 1 and 2, countries 2 and 3 and countries 1 and 3 respectively. The parameters τ_1, τ_2 and τ_3 (> 1) indicate the presence of Samuelson's iceberg-type transactions costs respectively between countries 1 and 2, countries 2 and 3 and countries 1 and 3. The consumer needs to buy τ units of foreign goods to consume 1 unit. When the two small countries form a currency union, they adopt the same currency $e = 1$ and the transaction costs disappear $\tau = 1$.

Y_1, Y_2 and Y_3 are the respective nominal incomes of countries 1, 2 and 3. In the absence of dividends paid to workers, wages constitute the only source of income. The nominal incomes are therefore equal to wages multiplied by the amount of labour:

$$Y_1 = W_1(L_A + L_{N1}) \leq W_1 L_1; Y_2 = W_2(L_B + L_{N2}) \leq W_2 L_2; Y_3 = W_3(L_C + L_{N3}) \leq W_3 L_3 \tag{6}$$

where the equalities hold in full employment equilibria.

The income of the representative consumer in country i is simply Y_i / L_i.

Solving for the optimal consumption by representative consumers and computing for total consumption of goods yields:

$$\begin{aligned}
\mathrm{A}^{\mathrm{d}} &= \alpha_1 \frac{Y_1}{P_A} + \alpha_2 \frac{Y_2}{P_A e_1} + \alpha_3 \frac{Y_3 e_3}{P_A} \\
\mathrm{B}^{\mathrm{d}} &= \beta_1 \frac{Y_1 e_1}{P_B} + \beta_2 \frac{Y_2}{P_B} + \beta_3 \frac{Y_3 e_2}{P_B} \\
\mathrm{C}^{\mathrm{d}} &= \gamma_1 \frac{Y_1}{P_C e_3} + \gamma_2 \frac{Y_2}{P_C e_2} + \gamma_3 \frac{Y_3}{P_C} \\
\mathrm{N}_1^d &= (1 - \alpha_1 - \beta_1 - \gamma_1) \frac{Y_1}{P_{N_1}} \\
\mathrm{N}_2^d &= (1 - \alpha_2 - \beta_2 - \gamma_2) \frac{Y_2}{P_{N_2}} \\
\mathrm{N}_3^d &= (1 - \alpha_3 - \beta_3 - \gamma_3) \frac{Y_3}{P_{N_3}}
\end{aligned} \tag{7}$$

Preference parameter α_i measures the share of country A's traded good in country i's consumption. The same interpretation holds for the other preference parameters.

Prices are assumed to be flexible. Equilibrium in each sector yields:

$$A = \alpha_1 \frac{Y_1}{P_A} + \alpha_2 \frac{T_e}{P_A e_1} + \alpha_3 \frac{Y_3 e_3}{P_A} = \theta_A L_A$$

$$B = \beta_1 \frac{Y_1 e_1}{P_B} + \beta_2 \frac{Y_2}{P_B} + \beta_3 \frac{Y_3 e_2}{P_B} = \theta_B L_B$$

$$C = \gamma_1 \frac{Y_1}{P_C e_3} + \gamma_2 \frac{Y_2}{P_C e_2} + \gamma_3 \frac{Y_3}{P_C} = \theta_C L_C \tag{8}$$

$$N_1 = (1 - \alpha_1 - \beta_1 - \gamma_1) \frac{Y_1}{P_{N_1}} = \theta_{N_1} L_{N_1}$$

$$N_2 = (1 - \alpha_2 - \beta_2 - \gamma_2) \frac{Y_2}{P_{N_2}} = \theta_{N_2} L_{N_2}$$

$$N_3 = (1 - \alpha_3 - \beta_3 - \gamma_3) \frac{Y_3}{P_{N_3}} = \theta_{N_3} L_{N_3}$$

Money, prices and exchange rates

Money is introduced by assuming a standard Cambridge equation, with a constant velocity.

Money market equilibrium in country *i* is hence assumed to be represented by

$$V_i M_i = Y_i . \tag{9}$$

where M_i and V_i represent respectively money supply and velocity in country *i*.

Trade balance equilibrium is reached through nominal exchange rate adjustment in flexible exchange rate regimes. Trade balance equilibrium in countries 1, 2 and 3 respectively is given by:

$$\begin{aligned} &\alpha_2 \cdot \frac{Y_2}{e_1} + \alpha_3 \cdot e_3 \cdot Y_3 - (\gamma_1 + \beta_1) Y_1 = 0 \\ &\beta_1 e_1 \cdot Y_1 + \beta_3 e_2 Y_3 - (\alpha_2 + \gamma_2) Y_2 = 0 \\ &\gamma_1 \frac{Y_1}{e_3} + \gamma_2 \frac{Y_2}{e_2} - (\alpha_3 + \beta_3) . Y_3 = 0 \end{aligned} \tag{10}$$

Goods market equilibrium and trade balance yield the equilibrium prices of goods:

$$P_A = \frac{(\alpha_1 + \beta_1 + \gamma_1) Y_1}{\theta_A L_A}; \; P_A = \frac{(\alpha_2 + \beta_2 + \gamma_2) Y_2}{\theta_B L_B}; \; P_C = \frac{(\alpha_3 + \beta_3 + \gamma_3) Y_3}{\theta_C L_C} \tag{11}$$

Money market equilibrium and trade balance yield the equilibrium level of exchange rates in a flexible exchange rate regime:

$$
\begin{aligned}
e_1 &= \frac{\alpha_2\beta_3 + \alpha_3(\alpha_2+\gamma_2)}{\alpha_3\beta_1 + \beta_3(\beta_1+\gamma_1)}\frac{V_2M_2}{V_1M_1} = s_1\frac{V_2M_2}{V_1M_1} \\
e_2 &= \frac{\gamma_2\beta_1 + \gamma_1(\alpha_2+\gamma_2)}{\alpha_3\beta_1 + \beta_3(\beta_1+\gamma_1)}\frac{V_2M_2}{V_3M_3} = s_2\frac{V_2M_2}{V_3M_3} \\
e_3 &= \frac{\gamma_2\beta_1 + \gamma_1(\alpha_2+\gamma_2)}{\alpha_2\beta_3 + \alpha_3(\alpha_2+\gamma_2)}\frac{V_1M_1}{V_3M_3} = s_3\frac{V_1M_1}{V_3M_3}
\end{aligned} \tag{12}
$$

Shocks

Preferences and monetary parameters are subject to shocks before consumers and firms make their optimal choice. Their percentage changes (denoted ^) are distributed as truncated normals, whose means and variances are in brackets:

$$
\begin{aligned}
\hat{\alpha}_i &\sim N(0, \sigma_{\alpha_i}) \\
\hat{\beta}_i &\sim N(0, \sigma_{\beta_i}) \\
\hat{\gamma}_i &\sim N(0, \sigma_{\gamma_i}) \\
\hat{V}_i &\sim N(0, \sigma_{V_i})
\end{aligned} \tag{13}
$$

A. Initial Equilibrium

1) *Firms' behaviour*

Firms behave competitively and face a labour supply curve which is infinitely elastic at the given wage until full employment is reached. National employment cannot rise above full employment. Hence, after the resolution of uncertainty, domestic and foreign firms maximise profits subject to, respectively:

$$
\begin{aligned}
w_1 &= w_{S1} \;\; ; \;\; L_A + L_{N1} \le L_1 \\
w_2 &= w_{S2} \;\; ; \;\; L_B + L_{N2} \le L_2 \\
w_3 &= w_{S3} \;\; ; \;\; L_C + L_{N3} \le L_3
\end{aligned} \tag{14}
$$

When the initial full-employment equilibrium is disturbed by an increase in demand for the goods of one country, firms cannot hire more workers to produce more, so that prices increase. When demand for the goods of one country goes down, in order to avoid losses (due to fixed wages), firms of that country will reduce employment until their aggregate output equals aggregate demand at the marginal cost pricing.

2) *Uncertainty and the timing of actions*

Uncertainty arises from goods demand and monetary shocks. We assume that the world is initially in full employment equilibrium. The corresponding nominal wages are respectively w_1, w_2 and w_3.

Before the uncertainty is resolved, nominal wages are set at levels, w_{s1}, w_{s2} and w_{s3} which are above the equilibrium wages w_1, w_2 and w_3. Such an assumption introduces nominal rigidities and allows for the incorporation of an inflationary bias as in Barro-Gordon (1983a, 1983b).

After the resolution of uncertainty, with wages given, unexpected demand and monetary shocks appear, and firms choose the competitively optimal employment under the constraint that the equilibrium wages are given by:

$$\hat{w}_i = \hat{M}_i + \hat{V}_i \text{ if } \hat{M}_i + \hat{V}_i \geq E(\hat{M}_i) \tag{15}$$

$\hat{w}_i = E(\hat{M}_i)$ if $\hat{M}_i + \hat{V}_i \leq E(\hat{M}_i)$ (downwards nominal rigidity of wages).

where $\hat{V}_i$, $\hat{M}_i$, $\hat{w}_i$ and $\hat{w}'_{si}$ denote respectively the growth rates of the money velocity, money stock, wages and minimum wage of country i, and $E(\hat{M}_i) = \mu_i$ where μ_i is the target inflation rate of the monetary authorities in country i. The equilibrium price is given by

$$P_j = \frac{V_i \cdot M_i}{\theta_i \cdot L_i} \text{ with } E(\theta_i) = 0 \tag{16}$$

B. The Authority's Loss Function

We define the authorihy's loss function with respect to inflation and unemployment in order to measure the net benefits that are expected to arise from the participation in a currency union or other kinds of exchange rate regimes. For instance, OCA theory proposes that the benefits of participating in a single currency result from the difference between the gains from the adoption of a single currency and the adjustment costs in terms of inflation and unemployment. The authority's loss function for inflation and unemployment used here are similar to the monetary authority's loss function used in macroeconomics since Barro-Gordon (1983a and 1983b). The square terms in our specification are superfluous because no trade-off exists between inflation and unemployment in the model.[3]

The authority's loss function of country i are assumed to be

$$H^i = E(u^i + \lambda^i \pi^i + CT^i) \tag{17}$$

where i indicates the country, E is the expectation operator, $u^i > 0$ is the unemployment rate, π^i is the inflation rate. In contrast with Ricci (1997), where the inflation rate is measured by the change in GDP-deflator, we assume that the inflation target is the increase in traded good prices.[4] λ is the relative weight the authority assigns to inflation versus employment. The loss function is measured as a percentage of the labor force (or equivalently as a percentage of full employment GDP, given the constant returns to scale assumption); as a consequence, transactions costs and unemployment have the same weight.

C. Transaction Costs

As specified in the model of Ricci (1997), the transaction costs are meant to represent all the additional deadweight and efficiency losses implied by multiple currencies. Due to the Samuelson's iceberg assumption, paying transaction costs is like wasting hours of work.

$$TC = (\beta_1 + \gamma_1)Y_1 - \left(\frac{\beta_1}{\tau_1} + \frac{\gamma_1}{\tau_3}\right)Y_1 \tag{18}$$

The country 1 spends $(\beta_1 + \gamma_1)Y_1$ on foreign goods, but its citizens effectively consume $\frac{\beta_1 Y_1}{\tau_1} + \frac{\gamma_1 Y_1}{\tau_3}$, the difference being due to the transaction costs. In equilibrium, $Y_1 = W_1 L_1$. The transaction costs faced by country 1 are calculated as the difference between the amount demanded and the amount consumed, as a percentage of the working population.

$$\frac{TC}{Y_1} = \beta_1(1 - \tau_1^{-1}) + \gamma_1(1 - \tau_3^{-1}) \tag{19}$$

[3]Recall that prices increase in case of a positive demand shock, and employment decreases in case of a negative demand shock. The model is as a kinked aggregate supply curve.

[4]The degree to which a country should target traded versus nontraded goods is certainly a major issue for open economy inflation targeting. A rationale for our assumption, which makes the analysis more tractable, is that if countries truly small and very open, then traded goods will make up a high proportion of the consumption bundle. We would like to thank the referees for this suggestion.

Since in full employment $Y_1 = W_1L_1$, then $\frac{TC}{W_1L_1} = \beta_1(1-\tau_1^{-1}) + \gamma_1(1-\tau_3^{-1})$ $= \beta_1 r_1 + \gamma_1 r_3$ (20)

where $r_1 = 1-\tau_1^{-1}$, and $r_3 = 1-\tau_3^{-1}$.
r_1 and r_3 represent respectively the transaction costs per unit of expenditure on goods produced in the countries 2 and 3.

Under flexible and fixed exchange rate regime transaction costs are:

$$TC_{FIX} = TC_{FLEX} = \beta_1 r_1 + \gamma_1 r_3 \quad (21)$$

However, in a currency union between the two small countries

$$TC_{MU} = \gamma_1 r_3 \quad (22)$$

III. Shocks and Adjustment

We analyse the consequences of the short run adjustment to shocks for unemployment and inflation, under different exchange rate regimes. Relative to Ricci, a larger array of possible exchange rate regimes is considered:

- a flexible exchange rate regime, in which all countries float against each other;
- a fixed exchange rate regime of the Bretton-Woods type (BW, Fix1) where the large country (Country 3) retains monetary autonomy;
- a fixed exchange rate regime of the European Monetary System type (EMS, Fix2): the two countries peg their bilateral exchange rate, one of them acting as the follower and the other being the leader;
- a monetary union between the two small countries (countries 1 and 2) which floats against the large country;
- A pseudo-exchange rate union with the large country (the two small countries in monetary union decide to fix their exchange rate with the large country)

Distinction can be made between hard fixes that cannot be changed and adjustable fixes or pegs such as Bretton-Woods. However, in this study, we have undertaken only fixed exchange rate regimes of BW and EMS types.

Unless otherwise specified, changes of variables are calculated from the initial equilibrium and are expressed in percentage terms.

A. Flexible Exchange Regime (Flex)

Under the flexible exchange rate regime, the money stock is exogenous (the money supply is controlled by the monetary authorities, $\hat{M} = \mu$), and the adjustment of the trade balance is due to the variation of the exchange rate. Given the hypotheses of a minimum wage ($\hat{W}_i = E(\hat{M}_i)$ if $\hat{M}_i + \hat{V}_i \leq E(\hat{M}_i)$) and maximum limit to labour supply, we can obtain the level of anticipated inflation and domestic unemployment.

Anticipated Inflation:

$$\pi_A = \hat{W}_1 - \hat{\theta}_A \Rightarrow \pi_A = \mu_1 + \hat{V}_1 - \hat{\theta}_A \text{ if } \hat{V}_1 \geq 0$$

$$\pi_A = \mu_1 - \theta_A \qquad \text{if } \hat{V}_1 < 0$$

$$E(\pi_A) = \int_{\theta_A}\int_{\hat{V}_1} \pi_A(\hat{V}_1, \hat{\theta}_A) f_v(\hat{V}_1) f_\theta(\hat{\theta}_A) d\hat{V}_1 d\hat{\theta}_A$$

$$E(\pi_A) = \frac{\sigma_{\hat{V}_1}}{\sqrt{2\pi}} + \mu_1 \tag{23}$$

Anticipated Unemployment **:** $E(u_j) = \dfrac{\sigma_{\hat{V}_j}}{\sqrt{2\pi}}$ (24)

For the purpose of assessing the net benefits, it is convenient to define here a loss function in inflation and unemployment under this regime.

Equations (17), (18), (23), and (24) imply that the expected losses for the home country under a flexible exchange rate regime (H_{FLEX}) are:

$$H_{FLEX} = E(u_j + \lambda\pi_A + TC) = \frac{\sigma_{V_1}}{\sqrt{2\pi}}(\lambda - 1) + \lambda\mu_1 + TC \tag{25}$$

B. Fixed Exchange Rate Regime of the BW Type (Fix1): the Large Country Retains Monetary Autonomy

The money supply is fixed in country 3 ($\hat{M}_3 = \mu_3$). Given that shocks can create an imbalance in the goods market, the two countries (1 and 2) must adjust. As the exchange rate is fixed, the adjustment takes place through the money supply:

$$\hat{M}_1 = \mu_3 - \hat{s}_3 + \hat{V}_3 - \hat{V}_1 \text{; hence}$$

$$\textit{Anticipated inflation}: E(\pi_A) = \mu_3 + \frac{\sqrt{\hat{V}_3^2 + \hat{s}_3^2}}{\sqrt{2\pi}} \tag{26}$$

$$\textit{Anticipated unemployment } E(u_j) = \frac{\sqrt{\hat{V}_3^2 + \hat{s}_3^2}}{\sqrt{2\pi}} \tag{27}$$

Equations (17), (18), (26), and (27) imply that the expected losses for the home country under a fixed exchange rate regime of the BW type ($H_{FIX\,1}$) are:

$$H_{FIX1} = E(u_j + \lambda\pi_A + TC) = \frac{\sqrt{\hat{v}_3^2 + \hat{s}_3^2}}{\sqrt{2\pi}}(\lambda + 1) + \lambda\mu_3 + TC \tag{28}$$

C. Fixed Exchange Rate Regime of EMS type: Floating Against the Large Country

Countries 1 and 2 agree on a fixed bilateral exchange rate system which floats against the currency of the large country, and imposes a coordinated monetary policy. We denote by $\bar{\mu} = \xi\mu_1 + (1-\xi)\mu_2$ the common monetary rule, where ξ is the degree of autonomy of country 1's monetary policy. In the case where $\xi = 1$, country 1 retains full monetary autonomy in the bilateral system, and country 2 has to keep the bilateral exchange rate fixed. Otherwise $\xi = 0$, and country 2 sets the course of monetary policy, which country 1 has to follow. Under this assumption:

$$\hat{M}_1 = \xi\mu_1 + (1-\xi)(\mu_2 + \hat{s}_1 + \hat{V}_2 - \hat{V}_1) = \bar{\mu} + (1-\xi)(\hat{s}_1 + \hat{V}_2 - \hat{V}_1)$$

$$\hat{M}_2 = \xi(\mu_1 - \hat{s}_1 - \hat{V}_2 + \hat{V}_1) + (1-\xi)\mu_2 = \bar{\mu} + \xi(\hat{s}_1 - \hat{V}_2 + \hat{V}_1)$$

and

$$E(\pi_A) = \bar{\mu} + \frac{\sqrt{(1-\xi)^2(\sigma_{\hat{s}_1}^2 + \sigma_{\hat{v}_2}^2) + \xi^2\sigma_{\hat{v}_1}^2}}{\sqrt{2\pi}} \tag{29}$$

$$E(u_j) = \frac{\sqrt{(1-\xi)^2(\sigma_{\hat{s}_1}^2 + \sigma_{\hat{v}_2}^2) + \xi^2\sigma_{\hat{v}_1}^2}}{\sqrt{2\pi}} \tag{30}$$

From equations (17), (18), (29), and (30), the expected losses for the home country in a fixed exchange regime of EMS type are (H_{FIX2}):

$$H_{FIX2} = \frac{\sqrt{(1-\xi)^2(\sigma^2_{\hat{s}_1} + \sigma^2_{\hat{v}_2}) + \xi^2 \sigma^2_{\hat{v}_1}}}{\sqrt{2\pi}}(\lambda + 1) + \lambda\bar{\mu} + TC \tag{31}$$

D. Monetary Union between Countries 1 and 2 (MU): Floating Against the Large Country

In this case, inflation and unemployment for country 1 are same as in the EMS regime. The difference between the two regimes results from transaction costs. So, the expected losses are:

$$H_{MU} = \frac{\sqrt{(1-\xi)^2(\sigma^2_{\hat{s}_1} + \sigma^2_{\hat{v}_2}) + \xi^2 \sigma^2_{\hat{v}_1}}}{\sqrt{2\pi}}(\lambda + 1) + \lambda\bar{\mu} \tag{32}$$

IV. Evaluation of Exchange Rate Regimes: a Cost-Benefit Analysis

We define the net benefits (NB) for the home country (country 1) as the difference between the expected losses under the different regimes enumerated above. Such expected losses are evaluated through the authority's loss function defined in Section II.

A. The Adjustment Costs Component

We discuss the net benefits resulting from the adjustment costs in terms of inflation and unemployment. Regimes are compared according to their ability to reduce inflation, to stabilise monetary and real shocks and to economise on transaction costs. The net benefits from the participation in a currency union compared to flexible exchange rate therefore are:

$$NB_{MU/FLEX} = \lambda(\mu_1 - \bar{\mu}) + (\lambda + 1)\left(\frac{\sigma_{\hat{V}_1} - \sqrt{(1-\xi)^2(\sigma^2_{\hat{s}_1} + \sigma^2_{\hat{V}_2}) + \xi^2 \sigma^2_{\hat{V}_1}}}{\sqrt{2\pi}}\right) + \beta_1 r_1 \tag{33}$$

The expression of the net benefits may differ for the two countries, indicating that the two countries may disagree, on purely economic grounds, about the adoption of a common currency. The two countries constitute an optimum currency area if both countries expect positive net gains from the creation of currency union.

1) Monetary Shocks

Recall that money demand shocks (shift in one country's money demand) can be interpreted as monetary shocks in general. We assumed that monetary authorities are not allowed to pursue discretionary policies that would enable them to counteract the shocks. Thus in this study, we have not considered monetary shocks due to international currency substitution.

If real shocks are absent or subject to full adjustment, the adjustment cost component due to monetary shocks is given by

$$NB_{MU/FLEX} = \frac{(\lambda+1)}{\sqrt{2\pi}}\left(\sigma_{\hat{V}_1} - \sqrt{(1-\xi^2)\sigma^2_{\hat{V}_2} + \xi^2\sigma^2_{\hat{V}_1}}\right) \tag{34}$$

This component is positive if the following condition is satisfied: $\sigma^2_{\hat{V}_1} > \frac{(1-\xi)}{(1+\xi)}\sigma^2_{\hat{V}_2}$ (35)

Under the flexible regime, the variability of nominal domestic income of country 1 is due only to domestic monetary shocks $(\sigma_{\hat{y}_1} = \sigma_{\hat{V}_1})$. In a monetary union between two small countries 1 and 2, both domestic and foreign monetary shocks affects domestic income only partially, depending on the degree of autonomous of domestic country.

$$\sigma^2_{\hat{y}_1} = (1-\xi)^2(\sigma^2_{\hat{V}_2}) + \xi^2\sigma^2_{\hat{V}_1} \tag{36}$$

Our interpretation of this component is different from that of Ricci (2008) because ha takes into account the degree of openness in the transmission of shocks, while our analysis centre on the degree of monetary autonomy in the currency union. To make this point clearer, we find it useful to discuss two cases in more depth.

If the home country is fully autonomous ($\xi = 1$) the effects of monetary union on this country disappear whereas country 2 bears all the losses. The difference between monetary union and the flexible regime results from the fact that in a currency union the monetary authority relinquishes the exchange rate as an internal instrument of adjustment to deal with real shocks but it also imports the monetary stability (or instability) of the other member of the monetary union. However, if country 1 retain the independence of its monetary policy under the currency union, its situation is the same as in the flexible regime and therefore $NB_{MU/FLEX} = 0$. However, if country 1 acts as a follower i.e. depends fully on country 2 ($\xi = 0$), the net benefit is positive if $\sigma^2_{\hat{V}_1} > \sigma^2_{\hat{V}_2}$.

In this case, our interpretation is the same as that of Ricci (2008) for the case when monetary shocks are positively correlated across the countries of a monetary union. Therefore, the country with higher monetary instability would gain stability from the creation of a currency union. In contrast to Ricci (2008), McKinnon's argument (1963) on openness is not brought into question.

The comparison between monetary union and Fix1 allows us to define under what conditions it is advantageous to anchor the domestic currency on an international currency. Indeed, when the home country belongs in a monetary union and its exchange rate is anchored to the large country, the stability of its economy depends on the variability of real and monetary shocks for the large country ($\sigma^2_{\hat{y}_1} = \sigma^2_{\hat{S}_3} + \sigma^2_{\hat{V}_3}$). The home country would gain from monetary union if $(1-\xi)^2(\sigma^2_{\hat{S}_1} + \sigma^2_{\hat{V}_2}) + \xi^2\sigma^2_{\hat{V}_1} > \sigma^2_{\hat{S}_3} + \sigma^2_{\hat{V}_3}$. (37)

Indeed,

$$NB_{MU/Fix1} = \frac{(\lambda+1)}{\sqrt{2\pi}}\left(\sqrt{\sigma^2_{\hat{S}_3} + \sigma^2_{\hat{V}_3}} - \sqrt{(1-\xi^2)(\sigma^2_{\hat{S}_1} + \sigma^2_{\hat{V}_2}) + \xi^2\sigma^2_{\hat{V}_1}}\right) \quad (38)$$

If country 1 is fully autonomous ($\xi = 1$), the net benefit component is positive if $\sigma^2_{\hat{V}_1} > \sigma^2_{\hat{V}_3}$. By anchoring its currency on that of the large country, the home country would gain monetary stability from a monetary union.

2) Real Shocks

If we neglect monetary shocks, the creation of a monetary union or the institution of a fixed exchange rate regime of the Bretton Woods type, relative to a flexible exchange rate regime, would generate costs. This confirms that a flexible exchange rate regime is a real shock absorber. Indeed, recent OCA literature argues that the case for fixed rates is strong when shocks are more symmetric (Bayoumi, 1994; Frankel, 2004)

$$NB_{MU/FLEX} = -\frac{(\lambda-1)}{\sqrt{2\pi}}(1-\xi)\sigma_{\hat{S}_1} \quad (39)$$

$$NB_{FIX1/FLEX} = -\frac{(\lambda-1)}{\sqrt{2\pi}}\sigma_{\hat{S}_3} \quad (40)$$

In a currency union this negative component diminishes with the degree of monetary autonomy (ξ) and rises with the variance of the trade shocks of the home country ($\sigma_{\hat{S}_1}$). In the Bretton Woods fixed regime, the negative component rises with the variance of the shocks of the large country.

If $(1-\xi)\sigma_{\hat{S}_1} > \sigma_{\hat{S}_3}$, it is beneficial for the home country to peg its bilateral exchange rate to the other small country and to the large country.

$$NB_{FIX/FLEX} - NB_{MU/FLEX} = NB_{FIX1/MU} = \frac{(\lambda+1)}{\sqrt{2\pi}}[(1-\xi)\sigma_{\hat{S}_1} - \sigma_{\hat{S}_3}] \quad (41)$$

If country 1 is fully autonomous in terms of monetary policy in the currency union, its net benefit is zero:

$$NB_{MU/FLEX} = 0. \quad (42)$$

This result confirms the argument that countries facing asymmetric real shocks would have high costs if they renounced the exchange rate as an instrument of adjustment. The flexible exchange rate regime would be a shock absorber with $\sigma_{\hat{y}_1} = 0$ in comparison with fixed exchange rate regimes:

- in the monetary union case $\sigma_{\hat{y}_1} = (1-\xi)\sigma_{\hat{S}_1}$ (43)

- in the Fix 1 case $\sigma_{\hat{y}_1} = \sigma_{\hat{S}_3}$ (44)

In addition, our result illustrates well the (n-1) problem because in a monetary union the country with monetary policy autonomy ensures the stability of its own economy even in the presence of real shocks: $\sigma_{\hat{y}_1} = 0$. (45)

On the other hand, if the real shocks were perfectly and positively correlated, and had equal standard deviations, in contrast to Ricci's result, the adjustment would imply an additional cost of a monetary union relative to a flexible exchange rate regime, although the negative component does not depend on $\sigma_{\alpha_2\beta_1}$ and $\sigma_{\gamma_1\gamma_2}$. Masson and Patillo (2004) show that the correlation of terms of trade shocks are higher among WAEMU countries than between them and WAMZ countries, or among WAMZ, suggesting that WAEMU forms a desirable currency area. Forming a larger currency area might dilute WAEMU's advantage in this regard.

B. The Inflationary Bias Component

The component of the net benefits due to the existence of an inflationary bias is given by:

$$NB_{MU/FLEX} = \lambda(1-\xi)(\mu_1 - \mu_2) = \lambda(\mu_1 - \bar{\mu}) \quad (46)$$

This component indicates a net benefit for country 1 if the union chooses an average rate of growth of money supply which is lower than that of country 1, in other words if $\mu_1 > \bar{\mu}$, where $\bar{\mu}$ is the monetary rule in the monetary union.

This component rises with λ, the relative weight assigned by the authority to inflation, and diminishes with ξ, the degree of monetary policy autonomy. That is, the more country 1 is autonomous($\xi = 1$), the lower is the gain. If country 1 is fully autonomous, the net benefit from participation in monetary union disappears.

In what conditions should the monetary union authority anchor its single currency on that of the large country?

If the monetary rule $\bar{\mu}$ in the currency union is higher than the average rate of growth of money supply of country 3 (μ_3) or if it is difficult to find a monetary rule in the currency union, it is advantageous to anchor the single currency to that of the large country.

C. Transactions Costs

As in Ricci (2008), the transaction costs are a proxy for the deadweight and efficiency losses associated with the existence of multiple currencies:

$$NB_{MU/FIX2} = \beta_1 r_1 . \tag{47}$$

The net benefit of country 1 from participation in a monetary union with country 2 compared to one of the other exchange rate regimes increases with β_1, the share of country 2-made traded goods in country 1's aggregate demand.

The net benefit from participation of the three in a monetary union (the large acts as leader) compared to the case where the two small countries participate in the union increases with(γ_1). The net benefit compared to other exchange rate regimes (Fix1, Fix2, Flex) increases with($\beta_1 + \gamma_1$), where γ_1 represents the share of country 3-made traded goods in country 1's aggregate demand.

V. Conclusion

This paper has investigated the circumstances under which it is beneficial for small countries in a currency union to peg their currency to a large one (euro zone for example). We develop a three-country theoretical model with two small countries and a large one and a wide array of fixed exchange rate regimes is considered (EMS or Bretton Woods).

We would like to underline some main results.

When the home country belongs in a monetary union and its exchange rate is anchored to the large country, the stability of its economy depends on the variability of real and monetary shocks for the large country. Furthermore, if the monetary rule $\bar{\mu}$ in the currency union is higher than the average rate of growth of money supply of country 3 (μ_3) or if it is difficult to find a monetary rule in the currency union, it is advantageous to anchor the single currency to that of the large country. This is the case of the CFA franc zone which has pegged its exchange rate to the French franc and then the euro, and imported credibility from the Bank of France and then the European Central Bank.

As in Ricci, the analysis of the inflationary bias component confirms the advantage of a nominal anchor which permits the reduction of inflation: the advantage of tying one's hands. Indeed, if the objective of low inflation announced by the monetary authority is not time-consistent, the high inflation country can reduce its inflation by pegging its exchange rate to a low inflation currency. This means that the monetary authorities anchor the exchange rate irrevocably. This is the case of regimes which fix the exchange rate by legal action, that is exchange rate unions (Bordo and Jonung, 2003), and gain from the credibility of the nominal anchor via what Willett (2000) calls a 'credibility effect'. Otherwise, the pressure to maintain a fixed exchange rate imposes an anti-inflationary bias on monetary policy which Willett refers to as a 'disciplinary effect'. It involves limits on the monetary financing of the deficit on the one hand, and on the lax refinancing of the banking sector by the central bank on the other hand (Combes and Veyrune, 2004).

An extension to the work presented here would be of a great interest: the addition of a fourth country (two small countries and two large countries) in order to investigate what type of anchor should be chosen (US dollar or euro).

Acknowledgements

The authors wish to thank participants at the conferences at IJAS Conference Harvard University (*USA*), Eight Mediterranean Social and Political Research, Florence and Montecatini Term (*Italia*), AFSE (*Paris, France*), and EEFS (*Sofia, Bulgaria*) for helpful comments. We also thank valuable comments from David Cobham. We are responsible for any remaining error.

Received 8 September 2009, Revised 15 July 2010, Accepted 28 July 2010

References

Barro, R. Gordon D. (1983a), "A Positive Theory of Monetary Policy in a Natural-Rate Model", *Journal of Political Economy*, Vol.91, pp. 589-610.

Barro, R. Gordon D. (1983b), "Rules, Discretion and Reputation in a Model of Monetary Policy", *Journal of Monetary Economics*, Vol. 12, pp. 101-121

Bayoumi, T. (1994), "A formal Model of Optimum Currency Areas", *IMF Staff Papers*, Vol. 41, pp. 537-554.

Bordo, M. D., Jonung L (2003), "The Future of EMU: What Does the History of Monetary Unions Tell Us?" in Capie, F. and G. Woods (Eds), *Monetary Unions*, Theory, history, public choice, Routledge, London.

Braga de Macedo, J. (1985), "Small Countries in Monetary Unions: a Two-Tier Model", *NBER Working Paper*, N1634.

Combes, J. L., Veyrune R. (2004), "Effet de Discipline et Effet de Crédibilit de l'ancrage nominal", *Revue d'Economie Financière*, Vol. 75, pp. 55-69.

Frankel, J. (2004), "Real Convergence and Euro adoption in Central and Eastern Europe: Trade and Business Cycle Correlation, Paper for Conference on Euro Adoption in the Accession Countries", pp. 1-22.

Masson, P., Patillo C. (2004), *The Monetary Geography of Africa*, in Brookings Institutions Press, Washington D.C

McKinnon, R. (1963), "Optimum Currency Areas", *American Economic Review*, Vol. 53, 717-725.

Ricci, L. A (2008), "A Model of an Optimum Currency area, Economics: The Open Access", *Open-Assessment E Journal,* Vol. 2 (8), pp. 1-31.

Willet, T. (2000), "International financial Markets as Sources of Crises and Discipline: the too much", *Too Late Hypothesis Essays in International Finance*, Vol. 218, pp. 1- 48.

Appendix

A. Abbreviations and acronyms

WAEMU, West African Economic and Monetary Union. Members (Benin, Burkina Faso, Côte d'Ivoire, Guinea- Bissau, Mali, Niger, Senegal, and Togo) use the CFA franc issued by their central bank, BCEAO.

WAMZ, West African Monetary Zone. Its prospective members include the Gambia, Ghana, Guinea, Nigeria and Sierra Leone.

ECOWAS, Economic Community of West African States. Founded in 1975, comprises those countries in WAEMU and WAMZ, plus Cape Verde and Liberia.

B. Deriving the anticipated inflation and unemployment under flexible

exchange rate regime (the *other deriving calculates can be providing upon request*).

Under the flexible exchange rate regime, the money stock is exogenous (the money supply is controlled by the monetary authorities, $\hat{M} = \mu$), and the adjustment of the trade balance is due to the variation of the exchange rate.

Anticipated inflation

$$\hat{M} = \mu, \quad \hat{M}_1 = \mu_1, \quad \hat{M}_2 = \mu_2, \quad \hat{M}_3 = \mu_3$$

$$\pi_A = \hat{W}_1 - \hat{\theta}_A \Rightarrow \pi_A = \mu_1 + \hat{V}_1 - \hat{\theta}_A \qquad \text{if } \hat{V}_1 \geq 0$$

$$\pi_A = \mu_1 - \hat{\theta}_A \qquad \text{if } \hat{V}_1 < 0$$

$$E(\pi_A) = E(\hat{M}_1 + \hat{V}_1 - \theta_A) = \mu_1 + E(\hat{V}_1 - \theta_A) \quad \text{if } \hat{V}_1 \geq 0$$
$$= E(\mu_1 - \theta_A) \qquad \text{if } \hat{V}_1 < 0$$

$$E(\pi_A) = \int_{\theta_A} \int_{\hat{V}_1} \pi_A(\hat{V}_1, \hat{\theta}_A) f_V(\hat{V}_1) f_\theta(\hat{\theta}_A) d\hat{V}_1 d\hat{\theta}_A$$

$$E(\pi_A) = \int_0^{+\infty} [\int_{\theta_A} (\mu_1 + \hat{V}_1 - \hat{\theta}_A) f_{\hat{\theta}_A}(\hat{\theta}_A) d\hat{\theta}_A] f_{v_1}(\hat{V}_1) d\hat{V}_1$$

$$E(\pi_A) = \mu_1 + \int_0^{-\infty} [\int_{\theta_A} (\hat{V}_1 - \hat{\theta}_A) f_{\theta_A}(\hat{\theta}_A) d\hat{\theta}_A] f_{v_1}(\hat{V}_1) d\hat{V}_1 \text{, hence}$$

$$E(\pi_A) = \mu_1 + \int_0^{+\infty} \int_{\theta_A} \hat{V}_1 f_{\theta_A}(\hat{\theta}_A) f_{v_1}(\hat{V}_1) d\hat{\theta}_A d\hat{V}_1 - \int_0^{+\infty} [\int_{\theta_A} \hat{\theta}_A f_{\theta_A}(\hat{\theta}_A) d\theta_A] d\hat{V}_1$$

Sine $\int_{\hat{\theta}_A} \hat{\theta} f_\theta(\hat{\theta}_A) d\theta_A = E(\hat{\theta}_A) = 0 \Rightarrow$

$$E(\pi_A) = \mu_1 + \int_0^{+\infty} \hat{V}_1 f_{v_1}(\hat{V}_1) d\hat{V}_1 \int f_{\theta_A}(\hat{\theta}_A) d\theta_A = \mu_1 + \int_0^{+\infty} \hat{V}_1 f_{v_1}(\hat{V}_1) d\hat{V}_1 \text{ hence}$$

$$E(\pi_A) = \frac{\sigma_{V_1}}{\sqrt{2\pi}} + \mu_1$$

Anticipated unemployment $(-\hat{L}_i)$

$$\hat{W}_i = \hat{M}_i + \hat{V}_i - \hat{L}_i$$

$$u_i = E(\hat{M}_i) - \hat{M}_i - \hat{V}_i \text{ if } \hat{M}_i + \hat{V}_i \leq E(\hat{M}_i)$$

$$u_i = 0 \quad \text{Otherwise}$$

Under flexible exchange rate $\hat{M}_1 = E(\hat{M}_1) = u_1$, $u_1 = -\hat{V}_1$ if $\hat{V}_1 > 0$
$u_1 = 0$ if $\hat{V}_1 < 0$

$$E(u_1) = \frac{\sigma_{V_1}}{\sqrt{2\pi}}.$$